Optical theorem for electromagnetic field scattering by dielectric structures and energy emission from the evanescent wave

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We present an optical theorem for evanescent (near field) electromagnetic wave scattering by a dielectric structure. The derivation is based on the formalism of angular spectrum wave amplitudes and block scattering matrix. The optical theorem shows that an energy flux is emitted in the direction of the evanescent wave decay upon scattering. The energy emission effect from an evanescent wave is illustrated in two examples of evanescent wave scattering, first, by the electrical dipole and, second, one-dimensional grating with linelike rulings. Within the latter example, we show that an emitted energy flux upon evanescent wave scattering can travel through a dielectric structure even if the structure has a forbidden gap in the transmission spectrum of incident propagating waves.

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I. INTRODUCTION

The present work is devoted to the problem of energy transformation between propagating (homogeneous) and evanescent (exponentially decaying or near-field) electromagnetic waves at scattering by dielectric structures. This problem was first discussed in the studies of total internal light reflection on the boundary of two dielectric media [1], to give evidence of the existence of an evanescent wave in the medium of lower refraction index. The interest in the problem has been stimulated nowadays by near-field optics development [2], where a scanning probe interacts with an optical near field, for example, a sample surface [3] or a waveguide [4]. The radiation of a propagating wave created by this interaction is collected in the far-field domain. The collected far-field contains subwavelength information about the sample surface or the mode profile of a waveguide, in the case of the above examples. A specific actual problem concerning the subwavelength resolution of spatial temperature distribution inside a heated biological object, obtained on the basis of an object thermal microwave radiation, arises in passive functional tomography of a human body [5]. For theoretical treatment, a probe and a scanned sample are represented in some cases [3,6–8] as electrical dipoles. A more detailed consideration of the interaction of a scanning probe with an evanescent wave is given in studies of the Mie scattering of an evanescent wave by a dielectric sphere [9], including the case of structural resonances in the dielectric sphere on a dielectric surface [10]. Mutual transformations of energy between propagating and evanescent waves goes on permanently in natural dense discrete disordered media, such as snow and ice [11], and artificial regular media, such as patterned multilayer photonic structures [12] and photonic crystals, in particular, at the formation of frequency transmission spectrum with a forbidden band gap [13] or at probing the near-field properties of a structure near a forbidden band gap [14].

The scattering of a plane propagating electromagnetic wave by a dielectric object is usually described in terms of a scattering amplitude, which characterizes the scattered electric field in the far wave zone of the object. According to the optical theorem [1], the total scattering cross section of an object is connected with an imaginary part of the scattering amplitude, taken in the forward scattering direction. A physical meaning of the optical theorem consists in that the power of an incident propagating wave undergoes extinction because of scattering.

We formulate an extended optical theorem for vector wave scattering, evanescent waves included, by a dielectric structure and show that a propagating wave created upon scattering of an evanescent wave gives rise to an energy flux (energy emission) in the direction of incident evanescent wave decay. This effect is similar to the cold emission of electrons from metals upon an applied electric field [15], which is a kind of tunneling. Note, in this connection, works [16] and [17] where various diffraction phenomena were treated as manifestations of tunneling. For example, a tunneling of electromagnetic radiation into cloud water droplets [18] or silica microspheres [19] contributes significantly to the meteorological glory or quality factor of whispering-gallery modes in optical microsphere resonators, respectively.

In Ref. [20], the energy emission effect from evanescent wave has been considered briefly. In this work, we consider this effect in detail from a theoretical point of view with different physical applications. Our derivation is based on the formalism of Sommerfeld-Weyl angular-spectrum decomposition of wave amplitudes (see, e.g., [21]) and the 2×2 block S-scattering matrix [22] of a dielectric structure. Unlike [22], we express the S-matrix blocks, that is, two reflection and two transmission operator coefficients, in terms of the scattering operator [23] (T-matrix formalism [24]) of the structure, following Ref. [25]. Using the optical

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theorem [24] (see also [26–28]) for the scattering operator of a dielectric lossless structure, we obtain an extended unitarity of the S matrix for the case of electromagnetic (vector) waves which generalizes the extended unitarity of the S matrix for scalar waves obtained in [22] within macroscopic consideration. Besides, our extended unitarity of the S matrix is written in a compact operator form with the aid of a projector technique and a block Pauli matrix. The extended optical theorem is a part of the extended unitarity of the S matrix including the transmission and reflection coefficients of a wave to be incident onto one side of a structure. Note here that the extended unitarity of the S matrix cannot be considered completely without the reciprocity relation for the S matrix, which we write in compact form starting with the reciprocity for the scattering operator (or for the Green's tensor function [29,30]), unlike that in [31] where the reciprocity relations for the reflection and transmission coefficients were derived by the direct use of Lorentz's reciprocity theorem. Some peculiarities of the effect of energy emission from an evanescent wave are considered in two examples of evanescent wave scattering, by an electrical dipole [three-dimensional] (3D pointlike scatterer) and by a one-dimensional diffraction grating with linelike rulings (2D pointlike scatterers). In the latter example, a linelike scatterer is defined as a dielectric cylinder in the limit when the cylinder diameter tends to zero, while the dielectric permittivity of it tends to infinity, so that their product becomes constant. A similar definition of a planelike scatterer (1D pointlike scatterer) is given in [32,33]. The matrix coefficients of wave transmission through and reflection from the 1D grating with linelike rulings are evaluated by an asymptotic solution of the Riccati and the associated equations [13,25]. These matrix transmission and reflection coefficients are applied to the problem of evanescent wave spatial spectroscopy based on the effect of energy emission from an evanescent wave. In accordance with two measuring methods known in the near-field optics [2], we consider two scenarios of evanescent wave spatial spectroscopy: (i) by varying the distance between the reference plane of the incident evanescent wave and the grating and (ii) by moving the grating along the reference plane. For this purpose, an evanescent incidence wave onto the 1D grating is specified as created by a planelike source, with the electric current density being parallel to the grating rulings. The electric current density in the form of a periodical array of linelike sources and periodically modulated white noise source is studied. Within these forms of the electrical current density we demonstrate an interference pattern in energy emission of evanescent waves through 1D grating with linelike rulings. Using the example of 1D grating with linelike rulings, we show that the energy flux emitted from an evanescent wave upon its scattering by a dielectric structure can travel through the structure even if the structure has a forbidden gap in the transmission spectrum for the incident propagating waves. In this connection we discuss some results of the experiment [14] where light interaction from a near-field probe with 3D photonic crystals near a forbidden band gap was investigated.

The organization of the paper is as follows. In Sec. II the tensor operator coefficients for transmission through and reflection from a dielectric structure of angular spectrum amplitudes of the electric field of an incident monochromatic electromagnetic wave are defined in terms of the scattering operator for the dielectric structure. The 2×2 block S-scattering matrix is introduced. The extended unitarity for the S matrix is formulated in compact operator form in Sec. III, using an optical theorem for the scattering operator of a dielectric structure. Here we also formulate the reciprocity relation for the S matrix. The extended optical theorem for the transmission and reflection coefficients of a dielectric structure is formulated in Sec. IV. In this section, the basic equation for the effect of energy emission from an evanescent wave upon its scattering by a dielectric structure is also derived. Section V presents two applications of the basic equation for the effect of energy emission from an evanescent wave upon its scattering, by electrical dipole and by 1D grating with linelike rulings. Conclusions are made in Sec. VI. Appendix A consists of an asymptotic solution of the Riccati and the associated equations for the matrix coefficients for transmission through and reflection from the 1D grating in the limit of linelike rulings. Appendix B gives some details on recurrent procedure to retrieve evanescent spectral orders.

II. SCATTERING MATRIX IN TERMS OF SCATTERING OPERATOR

A. Basic equations

We start with the Helmholtz vector wave equation for the electric field $\mathbf{E}(\mathbf{r})$ of a monochromatic electromagnetic wave in a 3D inhomogeneous isotropic dielectric structure writing the equation as

$$[\delta_{\alpha\beta}\nabla^2 - \nabla_\alpha\nabla_\beta + k_o^2 - V_{\alpha\beta}(\mathbf{r})]E_\beta = 0. \tag{1}$$

Here $k_o = \omega / C_o$ is the wave number in a background with the frequency ω, dielectric permittivity ε_o, and phase velocity C_o. An effective scattering tensor potential $V_{\alpha\beta}(\mathbf{r}) = V(\mathbf{r})\delta_{\alpha\beta}$ of the structure is defined by a scattering scalar potential $V(\mathbf{r}) = -k_o^2[\varepsilon(\mathbf{r}) - \varepsilon_o]/\varepsilon_o$ where $\varepsilon(\mathbf{r})$ is the structure dielectric permittivity. The summation over repeated Greek subscripts is implied in the limits from 1 to 3, with 1,2,3 corresponding to the x,y,z axes of the Cartesian coordinate system, respectively, and $\delta_{\alpha\beta}$ is the Kronecker symbol. The magnetic permeability is supposed to equal unity all over.

We are interested in the electric wave field $\mathbf{E}(\mathbf{r})$ outside (or inside) the dielectric structure, with the electric field of an incident electromagnetic wave denoted $\mathbf{E}^0(\mathbf{r})$. In terms of the Green tensor function $G^0_{\alpha\beta}(\mathbf{r})$ in the background and the scattering tensor operator $T_{\alpha\beta}(\mathbf{r},\mathbf{r}')$ of the structure, the electric wave field of interest can be written, using tensor operator denotations, as

$$E = E^0 + G^0 T E^0. \tag{2}$$

Note here that the Green's tensor function of the electric field in the background G^0 satisfies Helmholtz Eq. (1), without the scattering potential in the left-hand side (LHS) and with the delta-source term $\delta_{\alpha\beta}\delta(\mathbf{r})$ in the right-hand side (RHS) of the equation, and the scattering operator of electric field T satisfies the Lippman - Schwinger equation, $T = V + VG^0T$. In what follows, we transform basic Eq. (2) for the scattered electric field to represent it as the angular spectrum amplitudes.

B. Angular spectrum representation

Let a volume or a surface dielectric structure under consideration occupy a region between planes $z=0$ and $z=L$ and a monochromatic electromagnetic wave be incident onto the left boundary plane $z=0$ with the electric field $\mathbf{E}^0(\mathbf{r})$ being written as

$$\mathbf{E}^0(\mathbf{r}) = \int_{\mathbf{k}_\perp} \exp(i\mathbf{k}_\perp \cdot \mathbf{r}_\perp)\mathbf{E}^0(\mathbf{k}_\perp, z). \quad (3)$$

In the RHS of this equation the denotation $\int_{\mathbf{k}_\perp} = (2\pi)^{-2}\int d\mathbf{k}_\perp$ is used, with $\mathbf{k}_\perp$ being the component of a wave-vector $\mathbf{k}$ transverse to the z axis. The angular spectrum amplitude $\mathbf{E}^0(\mathbf{k}_\perp)$ of the incident electric field is defined by $\mathbf{E}^0(\mathbf{k}_\perp,z)=\mathbf{E}^0(\mathbf{k}_\perp)\exp(i\gamma_\mathbf{k} z)$ and describes either a propagating or an evanescent wave, depending on $k_\perp < k_o$ and $\gamma_k = \sqrt{k_o^2 - k_\perp^2}$ is real or $k_\perp > k_o$ and $\gamma_k = i\sqrt{k_\perp^2 - k_o^2}$ is a purely imaginary quantity, respectively. In angular spectrum representation [34] the Green's tensor function of the electric field in the background has the form

$$G^0_{\alpha\beta}(\mathbf{r}) = \mathcal{P}\int_{\mathbf{k}_\perp} P^{\mathrm{tr}}_{\alpha\beta}(\hat{\mathbf{k}}^\pm)\frac{1}{2i\gamma_k}\exp(i\mathbf{k}^\pm \mathbf{r}) + \frac{1}{k_o^2}\hat{z}_\alpha \hat{z}_\beta \delta(\mathbf{r}). \quad (4)$$

Here $\mathcal{P}$ denotes a principal part of the singular integral. The wave vectors of forward and backward going waves $\mathbf{k}^\pm = \mathbf{k}^\pm(\mathbf{k}_\perp)$ are defined by $\mathbf{k}^\pm = \mathbf{k}_\perp \pm \gamma_k \hat{z}$, respectively, with $\hat{z}$ being the unit vector along the z axis, and stand in the RHS of Eq. (4) for the cases of $z>0$ and $z<0$, respectively. The tensor $P^{\mathrm{tr}}_{\alpha\beta}(\hat{k}) = \delta_{\alpha\beta} - \hat{k}_\alpha \hat{k}_\beta$ denotes the orthogonal projector in the direction perpendicular to the unit vector $\hat{k}$ (transverse projector), $\hat{k}^2=1$, of the wave vector $\mathbf{k}$ that can be a complex one, the unit vectors $\hat{k}^\pm = \mathbf{k}^\pm / k_o$. Applying the angular spectrum representation to Eq. (2) and bearing in mind Eq. (4) gives

$$E_\alpha(\mathbf{k}_\perp, z) = E^0_\alpha(\mathbf{k}_\perp, z) + \exp(-i\gamma_k z)\int_{\mathbf{k}'_\perp} B_{\alpha\beta}(\mathbf{k}_\perp, \mathbf{k}'_\perp)E^0_\beta(\mathbf{k}'_\perp) \quad (5)$$

as $z<0$ and

$$E_\alpha(\mathbf{k}_\perp, z) = \exp(i\gamma_k z)\int_{\mathbf{k}'_\perp} A_{\alpha\beta}(\mathbf{k}_\perp, \mathbf{k}'_\perp)E^0_\beta(\mathbf{k}'_\perp) \quad (6)$$

as $z>L$. The tensor coefficients for transmission through $A_{\alpha\beta}(\mathbf{k}_\perp, \mathbf{k}'_\perp)$ and reflection from $B_{\alpha\beta}(\mathbf{k}_\perp, \mathbf{k}'_\perp)$ the structure of a generally inhomogeneous plane wave are given by

$$A_{\alpha\beta}(\mathbf{k}_\perp, \mathbf{k}'_\perp) = P^{\mathrm{tr}}_{\alpha\beta}(\hat{k}^+)\delta_{\mathbf{k}_\perp, \mathbf{k}'_\perp} + \frac{1}{2i\gamma_k} a_{\alpha\beta}(\hat{k}^+, \hat{k}^{+\prime}) \quad (7)$$

and

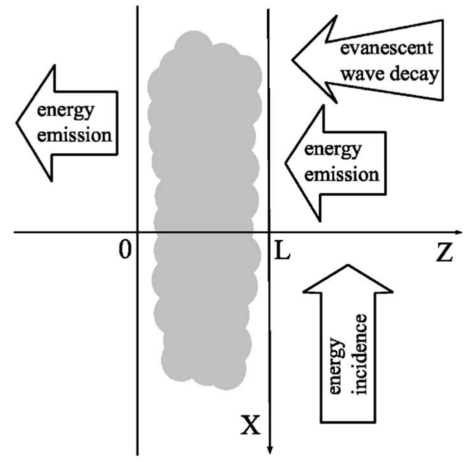

FIG. 1. A schematic presentation of a dielectric structure (gray area) between two boundary planes and the energy emission from an evanescent wave upon scattering by this structure.

$$B_{\alpha\beta}(\mathbf{k}_\perp, \mathbf{k}'_\perp) = \frac{1}{2i\gamma_k} a_{\alpha\beta}(\hat{k}^-, \hat{k}^{+\prime}), \quad (8)$$

where $\hat{k}^{\pm\prime}$ are the unit vectors along the vectors $\mathbf{k}^\pm(\mathbf{k}'_\perp)$ and $\delta_{\mathbf{k}_\perp, \mathbf{k}'_\perp}$ denotes $(2\pi)^2\delta(\mathbf{k}_\perp - \mathbf{k}'_\perp)$. The tensor $a_{\alpha\beta}(\hat{k}^\xi, \hat{k}^{\eta\prime})$, where $\xi, \eta = \pm$, is a scattering tensor amplitude of the structure, defined in the case of plane waves as follows:

$$a_{\alpha\beta}(\hat{k}^\xi, \hat{k}^{\eta\prime}) = P^{\mathrm{tr}}_{\alpha\mu}(\hat{k}^\xi)T_{\mu\nu}(\mathbf{k}_\perp, \xi\gamma_k; \mathbf{k}'_\perp, \eta\gamma_{k'})P^{\mathrm{tr}}_{\nu\beta}(\hat{k}^{\eta\prime}) \quad (9)$$

with $T_{\alpha\beta}(\mathbf{k}, \mathbf{k}')$ being the 3D spatial Fourier transform of the scattering operator $T_{\alpha\beta}(\mathbf{r}, \mathbf{r}')$ of electric field by the structure

$$T_{\alpha\beta}(\mathbf{k}, \mathbf{k}') = \int d\mathbf{r} \int d\mathbf{r}' \exp[-i(\mathbf{k}\cdot\mathbf{r} - \mathbf{k}'\cdot\mathbf{r}')]T_{\alpha\beta}(\mathbf{r}, \mathbf{r}'). \quad (10)$$

Remember that in the case of a discrete dielectric object [35], the scattering tensor amplitude (9) of an inhomogeneous wave generalizes the familiar scattering amplitude by an object [1] with the aid of analytical continuation into the complex domain of scattering angles, in accordance with the ideas of Sommerfeld and Weyl (see [21]), to consider both the scattered far field and near field.

An electromagnetic wave may be incident upon the right boundary plane $z=L$, as in Fig. 1, with an angular spectrum amplitude $\tilde{E}^0_\alpha(\mathbf{k}_\perp)$, which gives an incident electric field $\tilde{E}^0(\mathbf{r})$ if expression $\tilde{E}^0_\alpha(\mathbf{k}_\perp, z) = \tilde{E}^0_\alpha(\mathbf{k}_\perp)\exp(-i\gamma_k z)$ is substituted into an equation similar to Eq. (3). In this case, for the transmitted through, $\tilde{E}_\alpha(\mathbf{r})$ and reflected from the structure electric field $[\tilde{E}_\alpha(\mathbf{r}) - \tilde{E}^0_\alpha(\mathbf{r})]$ we find

$$\tilde{E}_\alpha(\mathbf{k}_\perp, z) = \exp(-i\gamma_k z)\int_{\mathbf{k}'_\perp} \tilde{A}_{\alpha\beta}(\mathbf{k}_\perp, \mathbf{k}'_\perp)E^0_\beta(\mathbf{k}'_\perp) \quad (11)$$

as $z<0$ and

$$\widetilde{E}_\alpha(\mathbf{k}_\perp,z)=\widetilde{E}^0_\alpha(\mathbf{k}_\perp,z)+\exp(i\gamma_k z)\int_{\mathbf{k}'_\perp}\widetilde{B}_{\alpha\beta}(\mathbf{k}_\perp,\mathbf{k}'_\perp)\widetilde{E}^0_\beta(\mathbf{k}'_\perp) \tag{12}$$

as $z>L$. Here the tensor coefficients of an inhomogeneous plane wave transmission through $\widetilde{A}_{\alpha\beta}(\mathbf{k}_\perp,\mathbf{k}'_\perp)$ and reflection from $\widetilde{B}_{\alpha\beta}(\mathbf{k}_\perp,\mathbf{k}'_\perp)$ the structure are given by

$$\widetilde{A}_{\alpha\beta}(\mathbf{k}_\perp,\mathbf{k}'_\perp)=P^{\rm tr}_{\alpha\beta}(\hat{k}^-)\delta_{\mathbf{k}_\perp,\mathbf{k}'_\perp}+\frac{1}{2i\gamma_k}a_{\alpha\beta}(\hat{k}^-,\hat{k}^{-\prime}) \tag{13}$$

and

$$\widetilde{B}_{\alpha\beta}(\mathbf{k}_\perp,\mathbf{k}'_\perp)=\frac{1}{2i\gamma_k}a_{\alpha\beta}(\hat{k}^+,\hat{k}^{-\prime}). \tag{14}$$

The 2×2 block S matrix of the structure is defined in terms of the above tensor coefficients of wave transmission through and reflection from the structure as follows:

$$S=\begin{pmatrix} A & \widetilde{B} \\ B & \widetilde{A}\end{pmatrix}. \tag{15}$$

The S matrix enables the calculation of angular amplitudes of the fields transmitted through $AE^0(\widetilde{A}\widetilde{E}^0)$ and reflected from the $BE^0(\widetilde{B}\widetilde{E}^0)$ structure, provided the angular amplitudes $E^0(\widetilde{E}^0)$ of the incident fields are known.

III. EXTENDED UNITARITY AND RECIPROCITY OF SCATTERING MATRIX

A. Optical theorem for the scattering operator in angular spectrum representation

In the lossless case, the scattering operator T of the electric field by a dielectric structure obeys an optical theorem [24] (see also [26–28]) that reads $T-T^\dagger=T^\dagger(G^0-G^{0\dagger})T$. Here the "dagger" superscript denotes a complex conjugate transpose of a tensor operator $F_{\alpha\beta}(\mathbf{r},\mathbf{r}')$, defined by $(F^\dagger)_{\alpha\beta}(\mathbf{r},\mathbf{r}')=F^*_{\beta\alpha}(\mathbf{r}',\mathbf{r})$, and the "star" superscript means a complex conjugate. The application of the 3D spatial Fourier transform (10) to the optical theorem for the scattering operator, with allowance for the angular spectrum representation (4) of the tensor Green's function in the background, gives

$$\begin{aligned}T_{\alpha\beta}(\mathbf{p},\mathbf{p}')-T^*_{\beta\alpha}(\mathbf{p}'^*,\mathbf{p}^*)&=\int_{\mathbf{k}_\perp;(k_\perp<k_o)}\frac{1}{2i\gamma_k}[P^{\rm tr}_{\kappa\gamma}(\hat{k}^-)T^*_{\gamma\alpha}(\mathbf{k}^-,\mathbf{p}^*)T_{\kappa\beta}(\mathbf{k}^-,\mathbf{p}')\\&\quad+P^{\rm tr}_{\kappa\gamma}(\hat{k}^+)T^*_{\gamma\alpha}(\mathbf{k}^+,\mathbf{p}^*)T_{\kappa\beta}(\mathbf{k}^+,\mathbf{p}')].\end{aligned} \tag{16}$$

Note that integration in the RHS of this identity goes on over the transverse to the z axis component $\mathbf{k}_\perp$ of the wave vector of propagating waves only, $k_\perp<k_o$. Nevertheless, the three-dimensional free momenta $\mathbf{p}$ and $\mathbf{p}'$ may be complex ones. Taking $\mathbf{p}$ and $\mathbf{p}'$ to be equal to the complex wave-vectors $\mathbf{k}^\pm$ of forward and backward inhomogeneous waves, Eq. (16) would give 16 separate tensor identities, with respect to the cases of propagating and evanescent waves in both $\mathbf{k}^+$ and $\mathbf{k}^-$ wave vectors. A principal result consists in converting this set of 16 tensor identities into a set of projections of one energy identity for the scattering matrix (15). This energy matrix identity, which we will also refer to as extended unitarity, can be written

$$(\mathbb{H}^{pr}\mathbb{S})^\dagger(\mathbb{H}^{pr}\mathbb{S})=\mathbb{H}^{pr}\mathbb{I}\mathbb{H}^{pr}-i[\mathbb{H}^{ev}\Sigma_x\mathbb{S}-(\mathbb{H}^{ev}\Sigma_x\mathbb{S})^\dagger]. \tag{17}$$

This extended unitarity is written for a renormalized version $\mathbb{S}$ of the S matrix (15) defined by

$$\mathbb{S}=\begin{pmatrix}\mathbb{A} & \widetilde{\mathbb{B}}\\ \mathbb{B} & \widetilde{\mathbb{A}}\end{pmatrix}\equiv\hat{\gamma}^{1/2}S\hat{\gamma}^{-1/2}, \tag{18}$$

where $\hat{\gamma}^{\pm1/2}=\mathrm{diag}(\gamma^{\pm1/2},\gamma^{\pm1/2})$ are diagonal block matrices with diagonal blocks equal to the tensor–operators $\gamma_k^{\pm1/2}\delta_{\alpha,\beta}\delta_{\mathbf{k}_\perp,\mathbf{k}'_\perp}$. The complex conjugate transpose of a block matrix in Eq. (17) is done as usual in block matrix algebra, e.g., for the $\mathbb{S}$ matrix we have, $(\mathbb{S}^\dagger)_{ij}=(\mathbb{S}_{ji})^\dagger$, where block indices $i,j=1,2$ and the complex conjugate transpose of a tensor operator $F_{\alpha\beta}(\mathbf{k}_\perp,\mathbf{k}'_\perp)$ is defined similar to the above case of a tensor operator with the kernel depending on 3D space position vectors. The symbols $\mathbb{H}^{\rm pr}$ and $\mathbb{H}^{\rm ev}$ denote projectors on propagating and evanescent waves, respectively. Specifically, the projector on propagating waves is a diagonal block matrix, $\mathbb{H}^{\rm pr}=\mathrm{diag}(H^{\rm pr},H^{\rm pr})$, with diagonal blocks equal to the tensor operator $H(k_o-k_\perp)\delta_{\alpha,\beta}\delta_{\mathbf{k}_\perp,\mathbf{k}'_\perp}$ where the Heaviside step function $H(x)=1$ as $x\geqslant 0$ and $H(x)=0$ as $x<0$. The projector $\mathbb{H}^{\rm ev}=\mathrm{diag}(H^{\rm ev},H^{\rm ev})$ on evanescent waves is obtained from the projector on propagating waves by replacing the Heaviside function $H(k_o-k_\perp)$ for propagating waves by the Heaviside function $H(k_\perp-k_o)$ for evanescent waves. A block identity matrix $\mathbb{I}=\mathrm{diag}(I,\widetilde{I})$, with I and $\widetilde{I}$ being the identity tensor operators $P^{\rm tr}_{\alpha\beta}(\hat{k}^\pm)\delta_{\mathbf{k}_\perp,\mathbf{k}'_\perp}$, respectively, transforming identically the transverse plane waves going forward and backward with the wave-vectors $\mathbf{k}^\pm$, which may be complex ones. We denote $\Sigma_x=\left(\begin{smallmatrix}0&1\\1&0\end{smallmatrix}\right)$ the 2×2 block matrix generalization of the usual Pauli matrix σ_x.

Make some rearrangement in the block order of the scattering matrix (18) taking

$$\mathbb{S}_x=\begin{pmatrix}\mathbb{B} & \widetilde{\mathbb{A}}\\ \mathbb{A} & \widetilde{\mathbb{B}}\end{pmatrix}=\Sigma_x\mathbb{S}. \tag{19}$$

A similar scattering matrix was used in Ref. [22], in the case of scalar waves. The extended unitarity (17) takes a more simple form in terms of the rearranged scattering matrix (19)

$$(\mathbb{H}^{\rm pr}\mathbb{S}_x)^\dagger(\mathbb{H}^{\rm pr}\mathbb{S}_x)=\mathbb{H}^{pr}\mathbb{I}\mathbb{H}^{\rm pr}-i[\mathbb{H}^{\rm ev}\mathbb{S}_x-(\mathbb{H}^{\rm ev}\mathbb{S}_x)^\dagger]. \tag{20}$$

B. Reciprocity of scattering matrix

The reciprocity of the scattering operator T of a dielectric structure reads $T_{\alpha\beta}(\mathbf{r},\mathbf{r}')=T_{\beta\alpha}(\mathbf{r}',\mathbf{r})$, which is a consequence from similar reciprocity [29,30] of the Green's tensor function G of the electric field in a dielectric structure and a relation, $T=V+VGV$. To formulate a corresponding property for the scattering matrix, denote $(F^R)_{\alpha\beta}(\mathbf{k}_\perp,\mathbf{k}'_\perp)=F_{\beta\alpha}(-\mathbf{k}'_\perp,-\mathbf{k}_\perp)$, where the superscript R means a reciprocity transformation of any tensor operator $F_{\alpha\beta}(\mathbf{k}_\perp,\mathbf{k}'_\perp)$. In terms of this denotation, the reciprocity of the renormalized scattering matrix (18) means

$$\mathbb{S}^R \equiv \begin{pmatrix} \mathbb{A}^R & \mathbb{B}^R \\ \tilde{\mathbb{B}}^R & \tilde{\mathbb{A}}^R \end{pmatrix} = \begin{pmatrix} \tilde{\mathbb{A}} & \mathbb{B} \\ \tilde{\mathbb{B}} & \mathbb{A} \end{pmatrix} = \Sigma_x \mathbb{S} \Sigma_x. \tag{21}$$

The first equality in Eq. (21) defines the reciprocity transformation of the renormalized scattering matrix; this definition can be rewritten as $(\mathbb{S}^R)_{ij}=(\mathbb{S}_{ji})^R$ where the block indices $i,j=1,2$. The obtained reciprocity (21), after having been written for each block separately, coincides with the result reported in [31].

The reciprocity (21) takes a more simple form in terms of the rearrangement scattering matrix (19) as can be seen from the relation

$$\mathbb{S}_x^R = \mathbb{S}_x. \tag{22}$$

Applying the last reciprocity to the extended unitarity (20), we obtain another form of the extended unitarity

$$(\mathbb{S}_x \mathbb{H}^{\mathrm{pr}})(\mathbb{S}_x \mathbb{H}^{\mathrm{pr}})^\dagger = \mathbb{H}^{\mathrm{pr}}\mathbb{I}\mathbb{H}^{\mathrm{pr}} - i[\mathbb{S}_x \mathbb{H}^{\mathrm{ev}} - (\mathbb{S}_x \mathbb{H}^{\mathrm{ev}})^\dagger]. \tag{23}$$

The two relations (20) and (23) mean a unitarity of the scattering matrix in the case without evanescent waves (see, e.g., [36]).

IV. EXTENDED OPTICAL THEOREM

The extended optical theorem is obtained as part of the extended unitarity (17) for the S matrix including the transmission and reflection coefficients of a wave to be incident upon the right side of the structure in the form

$$(H^{\mathrm{pr}}\tilde{\mathbb{A}})^\dagger (H^{\mathrm{pr}}\tilde{\mathbb{A}}) + (H^{\mathrm{pr}}\tilde{\mathbb{B}})^\dagger (H^{\mathrm{pr}}\tilde{\mathbb{B}}) = H^{\mathrm{pr}}\tilde{I}H^{\mathrm{pr}} - i[H^{\mathrm{ev}}\tilde{\mathbb{B}} - (H^{\mathrm{ev}}\tilde{\mathbb{B}})^\dagger]. \tag{24}$$

The projection of this relation on propagating waves gives a conventional optical theorem [1], written in terms of the transmission coefficient deviation from an identity operator $\Delta\tilde{\mathbb{A}}=\tilde{\mathbb{A}}-\tilde{I}$ and reflection coefficient $\tilde{\mathbb{B}}$. The physical meaning of the extended optical theorem (24) is disclosed through the expressions for a total energy flux $\bar{P}_z(z)$ along the z axis direction in the regions $z<0$ and $z>L$. This flux is given by integrating the z component of the Poynting's vector for the total electromagnetic field over the x,y plane. The comparison shows that energy fluxes $\bar{P}_z(z)$ in the regions on the left and right from the boundary planes of the structure have the same value in accordance with the optical theorem. For the case of an incident propagating wave, where the renormalized angular spectrum amplitude $\tilde{f}_\alpha(\mathbf{k}_\perp)=\gamma_k^{1/2}\tilde{E}_\alpha^0(\mathbf{k}_\perp)$ satisfies the condition $H^{\mathrm{pr}}\tilde{f}=\tilde{f}$, the energy flux along the z axis on both sides of the structure is written as a sum of an incident field energy flux and some antiflux, which gives rise to the extinction of the incident field energy flux because of scattering, according to equations

$$\frac{8\pi\omega}{c^2}\bar{P}_z(z<0) = \frac{8\pi\omega}{c^2}\bar{P}_z(z>L) = -(H^{\mathrm{pr}}\tilde{f},H^{\mathrm{pr}}\tilde{f}) + (H^{\mathrm{pr}}\tilde{\mathbb{B}}H^{\mathrm{pr}}\tilde{f},H^{\mathrm{pr}}\tilde{\mathbb{B}}H^{\mathrm{pr}}\tilde{f}). \tag{25}$$

We have used here the Gaussian system of units and written the second equality in terms of the scalar product for vector functions of $\mathbf{k}_\perp$ defined by, $(\tilde{f},\tilde{f}')=\int_{\mathbf{k}_\perp}\tilde{f}_\alpha(\mathbf{k}_\perp)\tilde{f}'^*_\alpha(\mathbf{k}_\perp)$. A different situation arises with an evanescent wave incidence, which angular spectrum amplitude satisfies the condition $H^{\mathrm{ev}}\tilde{f}=\tilde{f}$. In this case, the energy flux along the z axis is negative on both sides of the structure according to equations

$$\frac{8\pi\omega}{c^2}\bar{P}_z(z<0) = \frac{8\pi\omega}{c^2}\bar{P}_z(z>L) = -(H^{\mathrm{pr}}\Delta\tilde{\mathbb{A}}H^{\mathrm{ev}}\tilde{f},H^{\mathrm{pr}}\Delta\tilde{\mathbb{A}}H^{\mathrm{ev}}\tilde{f}). \tag{26}$$

This energy flux (emission) is created when an incident evanescent wave is scattered by a dielectric structure into a wave propagating in the direction of incident evanescent wave (amplitude) decay. Note that the energy flux of the incident evanescent wave itself propagates across the direction of evanescent wave exponential decay (see Fig. 1).

Consider two applications of basic Eq. (26) for the effect of energy emission from an evanescent wave upon its scattering by a dielectric structure.

V. APPLICATIONS FOR ENERGY EMISSION EFFECT

A. Energy emission from an evanescent wave upon its scattering by electrical dipole

Let a dielectric structure in the form of an electrical dipole (3D pointlike scatterer) be placed in the point $\mathbf{r}_1$. The scattering operator of the electrical dipole is given (see, e.g., [37]) by $T_{\alpha\beta}(\mathbf{k},\mathbf{k}')=\tilde{t}\delta_{\alpha\beta}\delta(\mathbf{r}-\mathbf{r}_1)\delta(\mathbf{r}'-\mathbf{r}_1)$. Here $\tilde{t}$ is the scattering amplitude ($\tilde{t}$ matrix in [37]), which satisfies the optical theorem, $Q_s=|\tilde{t}|^2/(6\pi)=-\mathrm{Im}\,\tilde{t}/k_o$, with Q_s being the scattering cross section of the dipole and Im denoting the imaginary part of the quantity. Substitution of the dipole scattering operator into definition (13) of the transmission coefficient and the last one into the RHS of Eq. (26) gives

$$\frac{8\pi\omega}{c^2}\bar{P}_z(z<0) = \frac{8\pi\omega}{c^2}\bar{P}_z(z>L) = -\frac{1}{2}k_oQ_s\sum_\alpha |\tilde{E}_\alpha^0(\mathbf{r}_1|ev)|^2, \tag{27}$$

where the incident electric field $\tilde{E}_\alpha^0(\mathbf{r}_1|ev)$ is of pure evanescent nature. The obtained result shows that the effect of energy emission from an evanescent wave upon its scattering by an electrical dipole enables one to obtain direct information concerning the intensity distribution inside the evanescent wave, using the dipole as a scanning probe. From the practical point of view, it is more appropriate to use a system of dipoles instead of one scanning dipole. Therefore, the second application for the emission Eq. (26) will be concerned with evanescent wave incidence upon a 1D diffraction grating with linelike rulings (2D pointlike scatterers).

B. Energy emission from evanescent wave upon its scattering by 1D diffraction grating with linelike rulings

Let an electromagnetic wave be scattered by a 1D diffraction grating, whose rulings in the form of, e.g., cylinders with the radius R, are parallel to the y axis and placed periodically along the x axis with a period Λ. Suppose that the grating occupies a region $0<z<h<L$ denoting $h=2R$. The wave, with electric field being parallel to the rulings (TE polarization), is incident upon a plane $z=L$ and has the form of a series in the grating spectral orders

$$\widetilde{E}_y^o(x,z) = \sum_\nu \exp\left[i\frac{2\pi\nu}{\Lambda}x - i\gamma(\nu)(z-L)\right]\widetilde{E}_y^o(\nu). \quad (28)$$

Here $\gamma(\nu)=\sqrt{k_o^2-(2\pi\nu/\Lambda)^2}$, with spectral orders $\nu=0,\pm1,\pm2,\ldots$, and the phases of the spectral orders are evaluated from the reference plane $z=L$. The renormalized matrix coefficients $\widetilde{\mathbb{A}}_{\mu,\nu}$ and $\widetilde{\mathbb{B}}_{\mu,\nu}$ for transformation of the spectral order ν into the spectral order μ at transmission through and reflection from the grating can be evaluated by solving the Riccati and the associated equations [13,25]. In the limit of a grating with linelike rulings, the width $2x_o(z)$ (section by plane z=const) of each single ruling tends to zero, while the dielectric permittivity ε of it tends to infinity, so that the quantity $a=\bar{x}_o(\varepsilon-\varepsilon_o)/\varepsilon_o$, where $\bar{x}_o=\pi R/4$ is the mean value of $x_o(z)$ with respect to z, tends to a constant. Also suppose the height h of a single ruling to become so small that the quantities $\mu x_o(z)/(\Lambda)$ and $\gamma(\mu)z$, with $0<z<h$, can be thought of as infinitesimally small which means $|\mu|\leq\mu_{\max}\ll\min(\Lambda/\bar{x}_o,\Lambda/h)$ and $k_oh\ll1$. In the limit of a grating with linelike rulings, the above-mentioned Riccati and the associated equations are asymptotically resolved (see details in Appendix A) and the matrix coefficients of transmission through and reflection from the grating take a simple analytic form

$$\widetilde{\mathbb{A}}_{\mu,\nu} = \delta_{\mu,\nu} + \widetilde{\mathbb{B}}_{\mu,\nu};$$

$$\widetilde{\mathbb{B}}_{\mu,\nu} = \frac{i}{2}\hat{b}\frac{1}{\gamma^{1/2}(\mu)\gamma^{1/2}(\nu)}. \quad (29)$$

The quantity $\hat{b}$ in the RHS of the last equation is defined by, $\hat{b}=2/(\ell-ig^2)$, with $1/\ell=k_o^2ah/\Lambda$ and $g^2=\Sigma_{(|\mu|\leq\mu_{\max})}1/\gamma(\mu)$. This quantity satisfies some analogy of optical theorem, $\mathrm{Im}\,\hat{b}=(1/2)|\hat{b}|^2\,\mathrm{Re}(g^2)$, where the real part Re of g^2 is positive, $\mathrm{Re}(g^2)>0$. Due to this analogy, the matrix coefficients (29) of transmission through and reflection from a grating with linelike rulings satisfy the extended optical theorem (24).

In a special case when $k_o\Lambda\gg1$ and evanescent spectral orders do not contribute to the sum for g^2 and the wave interaction between rulings is not important, $g^2\approx\Lambda/2$. Substituting this into the definition of $\hat{b}$ leads to a relation $1/\Lambda\hat{b}=1/4\beta_o-i/4$, with $\beta_o=(\pi/4)(\varepsilon/\varepsilon_o-1)(k_oR)^2\ll1$, showing that the quantity $\Lambda\hat{b}$ coincides in this case with $4ib_o^*$ where b_o is the monopole coefficient of electromagnetic wave scattering by a single cylinder, which in the Rayleigh scattering limit is expressed through the phase angle β_o by a simple way [38]. Therefore, in the general case the quantity $\hat{b}$ may be thought of as a cooperative scattering coefficient of a ruling of a grating.

First, apply the transmission and reflection matrix coefficients (29) to the extinction equations (25), which take the form

$$\frac{8\pi\omega}{c^2}\bar{P}_z(z<0) = \frac{8\pi\omega}{c^2}\bar{P}_z(z>L) = -\sum_{\mu_{pr}}\gamma(\mu)|\widetilde{E}_y^0(\mu|pr)|^2 + \frac{1}{2}\mathrm{Im}\,\hat{b}|\widetilde{E}_y^0(x=0,z=0|\mathrm{pr})|^2. \quad (30)$$

Here the incident electric field $E_y^0(x,z|\mathrm{pr})$ consists of propagating spectral orders $\mu=\mu_{\mathrm{pr}}$ only.

The cooperative scattering coefficient $\hat{b}$ can be of specific resonant property under the conditions

$$k_o\Lambda<2\pi, \quad \mathrm{Im}(g^2)+\ell=0, \quad \hat{b}=i2k_o, \quad (31)$$

when the period Λ of a grating is smaller than the wavelength in the background and the cooperative scattering coefficient becomes a purely imaginary quantity. The insertion of Eq. (31) into extinction Eqs. (30) shows that under resonance conditions a grating with linelike rulings does not transmit a propagating wave. This forbidden frequency in the radiation transmission spectra of a propagating incident wave through the 1D grating under consideration can be estimated roughly as (see the Appendix A) $k_o\Lambda/2\pi\approx1-2\beta_o^2/\pi^2$, and is similar to a forbidden frequency [39] in the case of a 2D grating consisting of 3D pointlike scalar scatterers. The width of a corresponding gap is estimated by $\Delta k_o/k_o\approx16\beta_o^3/\pi^3$. In the two last estimations the quantity β_o is taken at $k_o=2\pi/\Lambda$.

Turn now to basic Eq. (26) for the effect of energy emission, which in the case of a grating with linelike rulings leads to

$$\frac{8\pi\omega}{c^2}\bar{P}_z(z<0) = -\frac{1}{2}\mathrm{Im}\,\hat{b}|\widetilde{E}_y^0(x=0,z=0|\mathrm{ev})|^2, \quad (32)$$

where the incident electric field $E_y^0(x,z|\mathrm{ev})$ consists of evanescent spectral orders only.

To make the physical sense of expression (32) more clear, let ■ denote the minimum (lowest) evanescent wave spectral order ν, where $2\pi|\nu|/\Lambda>k_0$, by n_1, and $n_2=n_1+1$, $n_3=n_1+2$, …, all other higher spectral orders of evanescent waves. Take further $E_k=\widetilde{E}_y^0(n_k)+\widetilde{E}_y^0(-n_k)$, where $k=1,2,3,\ldots$, is an amplitude of a symmetrical function with respect to the variable x of an evanescent wave of the spectral order n_k and $\gamma_k=|\gamma(n_k)|$. Using these denotations we reduce the expression $I(L|\mathrm{ev})=|\widetilde{E}_y^0(x=0,z=0|\mathrm{ev})|^2$ to the form

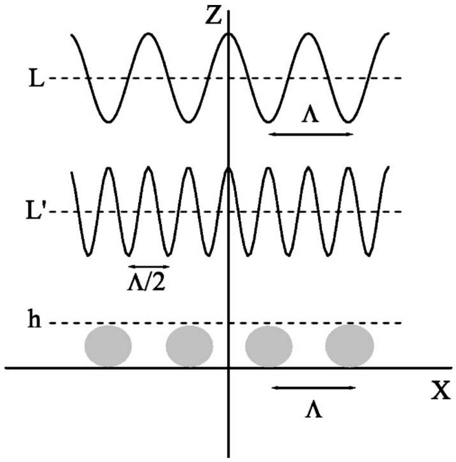

FIG. 2. Schematic drawing of the distant-spatial spectroscopy of an evanescent wave with a 1D grating of regularly (period Λ) situated cylinders ($h=2R$, R is cylinder radius) whose cross sections are shown as gray filled circles. The TE-polarized evanescent wave is incident upon the reference plane $z=L$ or $z=L'$. The lowest (period Λ) and first higher (period $\Lambda/2$) spectral orders are successively scanned by positioning the grating at the distances $L-h$ and $L'-h$ from the corresponding reference planes.

$$
\begin{aligned}
I(L|\mathrm{ev}) &= \sum_k \exp(-2\gamma_k L)|E_k|^2 \\
&\quad + \sum_{k'>k} \exp[-(\gamma_{k'}+\gamma_k)L]\Gamma(k',k) \\
&= \exp(-2\gamma_1 L)|E_1|^2 \\
&\quad + \exp[-(\gamma_2+\gamma_1)L]\Gamma(2,1) \\
&\quad + \exp(-2\gamma_2 L)|E_2|^2 + \cdots .
\end{aligned} \tag{33}
$$

Here a coefficient of interference $\Gamma(k',k)$ between evanescent spectral orders $n_{k'}$ and n_k is defined by $\Gamma(k',k)=E_{k'}E_k^* + E_{k'}^* E_k = 2|E_{k'}||E_k|\cos(\varphi_{k'}-\varphi_k)$, with $\varphi_{k'}$ and φ_k being phases of $E_{k'}$ and E_k, respectively. Second Eq. (33) is written in approximation of two evanescent spectral orders n_1 and n_2 in the series (28). Equation (33) becomes an asymptotic expansion in the limit $L\to\infty$. Bearing in mind a monotonous increase of γ_k with k, we derive from asymptotic expansion (33), first, the evanescent wave intensity $|E_1|^2$ of the lower spectral order n_1 and then the coefficient $\Gamma(2,1)$ of interference between this lower and the first higher spectral order n_2 of evanescent wave, if the function $I(L|\mathrm{ev})$ is known. The method of retrieving is based on a standard recurrent procedure in the theory of asymptotic expansions [40] and supposes, first, the use of the first term of expansion (33), followed by the subtraction of this term from $I(L|\mathrm{ev})$ and evaluation of the coefficient $\Gamma(2,1)$ of the second term of expansion (see details in Appendix B). This procedure which allows the retrieval of lower spectral orders of an evanescent wave by varying the distance between its reference plane $z=L$ and the grating $z=h$, may be called a distant-spatial spectroscopy of an evanescent wave and is depicted in Fig. 2. The higher the spectral order of an evanescent wave, the faster its amplitude decreases with the distance from the reference plane $z=L$. Hence, in particular, an evanescent wave of the lowest spectral order n_1 can be scanned with a grating at longer distance (for example, $L-h$ in Fig. 2) from the reference plane than an evanescent wave of the first higher spectral order n_2 (for example, $L'-h$ in Fig. 2).

Equation (33) can be a base not only for mentioned distant—spatial but also for an interference—spatial spectroscopy of evanescent waves. Really the form of the second Eq. (33) is similar to the well known interference law of two monochromatic waves in optics [1]. Such similarity means that the second Eq. (33) describes implicitly an interference pattern in energy emission of evanescent spectral orders n_1 and n_2 through 1D grating with linelike rulings. To disclose the interference pattern hidden in the second Eq. (33) we need a detailed consideration of the source construction of the incident evanescent wave. Besides, even the quantities $|E_k|^2$ describe an interference of two evanescent spectral orders propagating along the grating in opposite directions in accordance with definition of amplitudes E_k.

Returning to Eq. (28) we will note that this type of incident electric wave field may be created, for example, by a planelike source with an electric current density $\mathbf{j}(x,z)$ parallel to the y axis and confined inside a thin slab $L-\Delta L/2 < z < L+\Delta L/2$ with the ΔL thickness tending to zero and the current density tending to infinity, such that the product $j(x)=(4\pi\omega/ic^2)2\Delta L j_y(x,z)$ becomes constant. Supposing the planelike source $j(x)$ to be periodically varying along the x axis according to the expansion

$$
j(x) = \sum_\nu \exp\left(i\frac{2\pi\nu}{\Lambda}x\right) j_\nu \tag{34}
$$

and bearing in mind that the incident electric field $\tilde{E}_y^0(x,z)$ in Eq. (28) is expressed through the current density $j_y(x,z)$ with the aid of the scalar Green's function in the background, we can obtain the following relation:

$$
\tilde{E}_y^o(\nu) = \frac{1}{2i\gamma(\nu)} j_\nu . \tag{35}
$$

In what follows, we consider two specific cases for the planelike source $j(x)$.

C. Periodical array of linelike sources

Let the planelike source $j(x)$ have the form of a periodical array of linelike sources according to the representation

$$
j(x) = j_o \Lambda \sum_\nu \delta(x - x_\nu), \tag{36}
$$

where $x_\nu=\nu\Lambda-(\varphi_o/2\pi)\Lambda$ are the points of the x axis intersection by linelike sources and j_o is a positive quantity. The quantity φ_o in the expression for x_ν defines a detuning between the positions of linelike sources in the plane $z=L$ and positions of the linelike rulings of a 1D grating (see the inset in Fig. 3). Substituting Eq. (36) into Eq. (34) gives j_ν, which after the insertion into relations (35) leads to

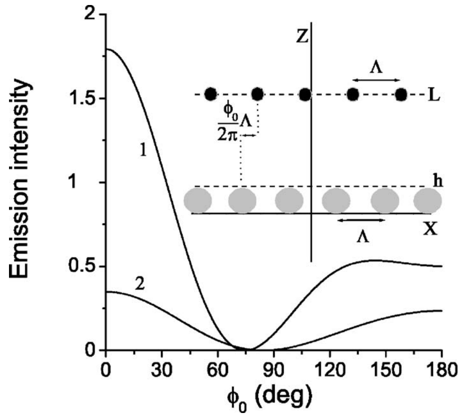

FIG. 3. Solid curves are the results of numerical calculations of the dimensionless emission intensity $I(L|\mathrm{ev})k_o^2/j_o^2$ [Eq. (38)] versus detuning φ_o between linelike sources and linelike rulings of the 1D grating (see the inset). The values of the parameters are taken from Ref. [14]: $\lambda_o = 2\pi/k_o = 647$ nm, $\Lambda = 482$ nm, $L-h = 10$ nm (curve 1), and 84 nm (curve 2). The inset schematically presents investigation of the energy emission effect from an evanescent wave generated by a periodical array of linelike sources (black filled circles) in the plane $z=L$. The linelike sources and rulings of the 1D grating (gray filled circles) have a detuning $(\varphi_o/2\pi)\Lambda$.

$$\widetilde{E}_y^0(\nu) = \frac{1}{2i\gamma(\nu)} j_o \exp(i\nu\varphi_o). \tag{37}$$

The evaluation of the quantities $|E_{1,2}|^2$ and $\Gamma(2,1)$ enables us to specify the second Eq. (33). As a result, the energy emission from an evanescent wave (32) becomes periodically varying with a detuning parameter φ_o, by moving the grating along the plane of sources, according to expression

$$I(L|ev) = j_o^2[d_1 \cos n_1\varphi_o + d_2 \cos n_2\varphi_o]^2 + \cdots, \tag{38}$$

where $d_k = \exp(-\gamma_k L)/\gamma_k$ and $k=1,2$. The behavior of the total expression in the RHS of Eq. (38) for different values of the detuning parameter φ_o is depicted in Fig. 3. It is seen that at an exact tuning, $\varphi_o = 0$, when the linelike sources are positioned exactly under the linelike rulings of the 1D grating, the emission intensity has a maximum. This maximum is defined by the equation $I(L|\mathrm{ev}) = j_o^2(d_1+d_2)^2$. In a detuning case, when $\phi_o = \pi$ and the linelike sources are positioned exactly between the linelike rulings, the emission intensity decreases and is defined by the equation $I(L|\mathrm{ev}) = j_o^2(d_1 - d_2)^2$. This theoretical result is in agreement with an experimental result of Ref. [14] where more light transmitted through a 3D photonic crystal was detected when the near field probe tip was positioned on the top of a crystal sphere. However, less light was detected when the tip was positioned between the spheres of the crystal. Note, that Eq. (38) predicts a zero emission intensity at some special detuning (see Fig. 3) rather than at $\phi_o = \pi$.

D. Periodically modulated white noise source

Consider now a planelike source of thermal electromagnetic radiation in the form of infinitesimally thin slab mentioned above, with the conductivity σ tending to infinity, such that the product $\Delta L\sigma$ becomes constant. The planelike source $j(x)$ may be chosen in this case to be periodically modulated along the x axis by a spatial white noise $\xi(x)$, that is, $j(x) = F(x)\xi(x)$. Here $F(x)$ is a periodic deterministic function given by a series in the grating spectral orders

$$F(x) = \sum_\nu \exp\left(i\frac{2\pi\nu}{\Lambda}x\right)F_\nu \tag{39}$$

and $\xi(x)$ is a stationary random spatial process

$$\xi(x) = \sum_\nu \exp\left(i\frac{2\pi\nu}{\Lambda}x\right)\xi_\nu \tag{40}$$

with noncorrelated random coefficients, $\langle \xi_\nu \xi_{\nu'}^* \rangle = \langle |\xi|^2 \rangle \delta_{\nu\nu'}$. A correlation function of the planelike source is given according to the two last equations by

$$\langle j(x) j^*(x') \rangle = |F(x)|^2 \langle |\xi|^2 \rangle \Lambda \delta_{\mathrm{per}}(x-x'), \tag{41}$$

where $\delta_{\mathrm{per}}(x)$ denotes a periodic δ function coinciding with the sum of δ functions in the RHS of Eq. (36) at $\varphi_o = 0$. Following the fluctuation theory [41] of thermal electromagnetic radiation, we suppose $|F(x)|^2$ to be a specification of the radiating slab temperature $T(x)$ periodically varying along the x axis around the mean temperature T_o as in the expansion

$$|F(x)|^2 = \frac{T(x)}{T_o} = \sum_\nu \exp\left(i\frac{2\pi\nu}{\Lambda}x\right)\hat{F}_\nu \tag{42}$$

with $\hat{F}_\nu = \Sigma_\mu F_\mu F_{\mu-\nu}^* = \hat{F}_{-\nu}^*$. Simultaneously the mean intensity $\langle |\xi|^2 \rangle$ of thermal fluctuation inside the radiating slab is supposed to be proportional to the product $T_o \Delta L\sigma$. The correlation function (41) of the source fluctuations leads to the following correlation matrix of spectral amplitudes (35) of an electric field (28) created by a planelike source of electromagnetic thermal radiation

$$\langle \widetilde{E}_y^o(\nu) \widetilde{E}_y^{o*}(\nu') \rangle = \frac{1}{4\gamma(\nu)\gamma^*(\nu')} \langle |\xi|^2 \rangle \hat{F}_{\nu-\nu'}. \tag{43}$$

Turn to the expression in the second Eq. (33) for the energy emission (32) from an evanescent wave (28) upon its scattering by a diffraction grating with linelike rulings. In the case of an incident evanescent wave created by a planelike source of thermal electromagnetic radiation, averaging of the expression mentioned over the ensemble of thermal fluctuations based in Eq. (43) gives

$$\begin{aligned} \langle I(L|ev) \rangle = \frac{1}{2} \langle |\xi|^2 \rangle [& d_1^2(1 + \mathrm{Re}\,\hat{F}_{2n_1}) \\ & + 2d_1 d_2(\mathrm{Re}\,\hat{F}_{n_2-n_1} + \mathrm{Re}\,\hat{F}_{n_1+n_2}) \\ & + d_2^2(1 + \mathrm{Re}\,\hat{F}_{2n_2})] + \cdots, \end{aligned} \tag{44}$$

where $\hat{F}_o = 1$. We see that this averaged expression enables the retrieval of the amplitude $\mathrm{Re}\,\hat{F}_{2n_1}$ of a symmetrical function with respect to the x variable harmonics of the temperature spatial distribution (42), with the spatial frequency

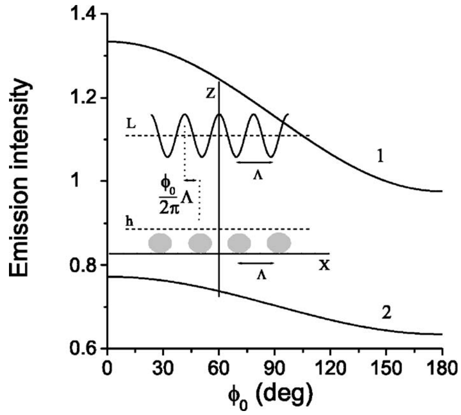

FIG. 4. Solid curves are the results of numerical calculations of the dimensionless emission intensity $\langle I(L|ev)\rangle 2k_o^2/\langle|\xi|^2\rangle$ [Eq. (45)] versus detuning φ_o between position of the maximums of the first cosine harmonics of planelike source of thermal radiation and linelike rulings of the 1D grating (see the inset). The values of the parameters are taken according to Ref. [5]: $\lambda_o=10$ cm, $\Lambda=8$ cm, $L-h=0.5$ cm(curve 1) and 1 cm (curve 2); $2|\hat{F}_1|=0.8$. The inset schematically presents an investigation of the energy emission effect from an evanescent wave generated by the planelike source of electromagnetic thermal radiation (the solid line represents the first cosine harmonics) in the plane $z=L$. The position of the harmonics maximums and the linelike rulings of the 1D grating (gray filled circles) have a detuning $(\varphi_o/2\pi)\Lambda$.

$4\pi n_1/\Lambda > k_o$ by varying the distance between the reference plane $z=L$ and the grating $z=h$. It is then possible to retrieve a sum, $\mathrm{Re}\,\hat{F}_{n_2-n_1}+\mathrm{Re}\,\hat{F}_{n_1+n_2}$, of amplitudes of the temperature symmetrical harmonics with spatial frequencies, $2\pi/\Lambda$, which may be less than k_o, and $2\pi(n_1+n_2)/\Lambda > k_o$.

To disclose an interference pattern hidden in Eq. (44) for the averaged energy emission (32) in approximation of two evanescent spectral orders n_1 and n_2 in series (28), we will specify the temperature distribution (42) by putting $\hat{F}_\nu=|\hat{F}_\nu|\exp(i\nu\varphi_o)$. In this case the radiating slab temperature is expanded in a series along cosine harmonics $2|\hat{F}_\nu|\cos[(2\pi\nu/\Lambda)(x+\varphi_o\Lambda/2\pi)]$ with amplitudes $2|\hat{F}_\nu|$ and detuning φ_o between positions of the cosine maximums and the linelike ruling of the 1D grating (see the inset in Fig. 4). We specify further the two considered evanescent spectral orders in series (28) to be $n_1=1$ and $n_2=2$ and take into account only the first cosine harmonics of order $\nu=1$ with amplitude $2|\hat{F}_1|$ of the radiating slab temperature distribution. After these simplifications Eq. (44) takes the form

$$\langle I(L|\mathrm{ev})\rangle = \frac{1}{2}\langle|\xi|^2\rangle(d_1^2+d_2^2+2d_1d_2|\hat{F}_1|\cos\varphi_o), \qquad (45)$$

where an apparent condition $2|\hat{F}_1|<1$ should be held. This simplest interference pattern for averaged energy emission through the 1D grating with linelike rulings from evanescent waves created by the planelike source of electromagnetic thermal radiation is depicted in Fig. 4. Equation (45) described this interference pattern is similar to that known in optics [1] as the interference law of two coherent in part light beams, with $|\hat{F}_1|$ being a degree of coherence of two evanescent spectral orders.

E. Separation of the energy emission effect

Note that when an incident electric field (28) consists of both propagating and evanescent spectral orders, the energy flux transmitted through the grating includes an extra cross term in Eqs. (30) and (32). This extra term is proportional, in the case of a grating with linelike rulings, to the real part $\mathrm{Re}\,\hat{b}$ of the cooperative scattering coefficient $\hat{b}$ and describes an interference between the contributions of propagating and evanescent spectral orders to the energy flux transmitted through the grating according to

$$\frac{8\pi\omega}{c^2}\bar{P}_z(z<0) = \mathrm{Re}\,\hat{b}\,\mathrm{Im}[\tilde{E}_y^{0*}(x=0,z=0|\mathrm{pr})$$
$$\times\tilde{E}_y^{0}(x=0,z=0|\mathrm{ev})]. \qquad (46)$$

In what follows, we suppose the period Λ of grating to be smaller than the wavelength in the background $k_o\Lambda<2\pi$ when there is only one propagating spectral order $\mu_{\mathrm{pr}}=0$ and all another spectral orders $n_1=1, n_2=2, n_3=3, \ldots$, are related to evanescent waves. In this case, the expression $I(L|\mathrm{pr},\mathrm{ev})=\mathrm{Im}[E_y^{0*}(x=0,z=0|\mathrm{pr})E_y^0(x=0,z=0|\mathrm{ev})]$ takes the form

$$I(L|\mathrm{pr},\mathrm{ev}) = \sum_k \exp(-\gamma_k L)\mathrm{Im}[\exp(-ik_oL)\tilde{E}_y^{0*}(\mu_{\mathrm{pr}}=0)E_k]. \qquad (47)$$

This expression becomes an asymptotic expansion under the conditions (37) of a periodical array of linelike sources

$$I(L|\mathrm{pr},\mathrm{ev}) = -\frac{1}{2k_o}j_o^2\cos(k_oL)\sum_k d_k\cos k\phi_o. \qquad (48)$$

Under conditions (43) of a periodically modulated white noise source, the averaging expression (47) over the ensemble of thermal fluctuations gives

$$\langle I(L|\mathrm{pr},\mathrm{ev})\rangle = -\frac{1}{2k_o}\langle|\xi|^2\rangle\cos(k_oL)\sum_k d_k\,\mathrm{Re}\,\hat{F}_{n_k}, \qquad (49)$$

where $\mathrm{Re}\,\hat{F}_1$ and $\mathrm{Re}\,\hat{F}_2$ are amplitudes of symmetrical functions with respect to the x variable harmonics of the temperature spatial distribution (42), with the spatial frequencies $2\pi/\Lambda > k_o$ and $4\pi/\Lambda > k_o$.

Compare expressions (30), (32), and (46) for energy fluxes transmitted through a grating with linelike rulings and caused by the contributions of propagating, evanescent spectral orders of an incident electric field (28) to these fluxes and by the interference between these contributions, respectively. Contribution (30) of propagating spectral orders can be separated because this contribution does not tend to zero exponentially fast with an increasing distance between the source plane $z=L$ of an incident electric field and a grating $z=h$. The more difficult problem is to separate the energy emission effect (32) caused by the contribution of evanescent incident spectral orders from the energy flux (46) caused by the interference between the contributions of propagating and evanescent incident spectral orders. The radical solution to the problem of energy emission separation is to use the resonant conditions (31), under which the real part of the cooperative scattering coefficient $\hat{b}$ tends to zero and the energy flux transmitted through a grating with linelike rulings consists of the contribution of evanescent spectral orders of an incident electric field alone.

VI. SUMMARY AND CONCLUSIONS

In this paper we have derived a basic equation for the effect of energy emission from an evanescent wave when scattered by a dielectric structure. Starting with a derived basic equation and using 1D grating with linelike rulings as a dielectric structure, we considered the distant-spatial and interference-spatial spectroscopy of evanescent waves. Both these scenarios of evanescent wave spatial spectroscopy were illustrated on examples of evanescent waves created by a planelike source with electrical current density being parallel to rulings of the grating. These examples were related to the problem to access optical details within the unit cell of a photonic crystal beyond the diffraction limit and to the problem of retrieving lower subwavelength spatial harmonics of temperature distribution along a planelike heated object, using the object thermal radiation. A serious problem arises concerning the separation of the transmitted (through a structure) energy flux coming from evanescent incident spectral orders from the flux caused by the interference of incident propagating and evanescent wave spectral orders. The radical solution to this problem is obtained by applying the 1D grating with line rulings and with a forbidden gap in the spectrum of radiation transmission for the incident propagating spectral order when the energy flux transmitted through the grating consists of contribution of evanescent spectral orders in the incident electric field only.

ACKNOWLEDGMENTS

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APPENDIX A: ASYMPTOTIC SOLUTION TO MATRIX RICCATI EQUATION FOR 1D GRATING WITH LINELIKE RULINGS

The renormalized matrix coefficients $\widetilde{\mathbb{A}}_{\mu,\nu}$ and $\widetilde{\mathbb{B}}_{\mu,\nu}$ for the transformation of the spectral order ν into the spectral order μ at the transmission through and reflection from the 1D grating considered in Sec. V B satisfy the matrix Riccati and the associated equations [13,25] in the form

$$\frac{d\widetilde{\mathbb{B}}}{dz} = \frac{V_o}{2\pi i}(U^{-1} + \widetilde{\mathbb{B}}U)\hat{M}(U^{-1} + U\widetilde{\mathbb{B}}) \tag{A1}$$

and

$$\frac{d\widetilde{\mathbb{A}}}{dz} = \frac{V_o}{2\pi i}\widetilde{\mathbb{A}}U\hat{M}(U^{-1} + U\widetilde{\mathbb{B}}) \tag{A2}$$

as $0<z<h$ and the “initial” conditions, $\widetilde{\mathbb{B}}(z=0)=0$ and $\widetilde{\mathbb{A}}(z=0)=\widetilde{I}$ where $\widetilde{I}$ is an identical matrix $\delta_{\mu\nu}$. The seeking matrix reflection and transmission coefficients of the grating are given by solutions of Eqs. (A1) and (A2), respectively, taken at $z=h$. In these two equations V_o denotes the scattering potential of a single cylindrical ruling and a diagonal matrix of free “evolution” $U_{\mu\nu}=\exp[i\gamma(\mu)z]\delta_{\mu\nu}$. The matrix $\hat{M}$ describes mutual transformations of both propagating and evanescent spectral orders because of their multiple scattering on the grating rulings. This transformation matrix is defined by $\hat{M}_{\mu\nu}=\gamma^{-1/2}(\mu)M_{\mu-\nu}\gamma^{-1/2}(\nu)$ where

$$M_\mu = \frac{1}{\mu}\sin\left[\frac{2\pi\mu}{\Lambda}x_o(z)\right]. \tag{A3}$$

In the limit of a grating with linelike rulings the last function tends to its limit form, $M_\mu \to 2\pi x_o(z)/\Lambda$. We also replace the diagonal matrix $U(z)$ and its inverse $U^{-1}(z)$ by the identity matrix $\widetilde{I}$ in the limit under consideration. As a consequence, the system of Eqs. (A1) and (A2) is transformed into the following limit form:

$$\frac{d\widetilde{\mathbb{B}}}{dz} = v(\widetilde{I} + \widetilde{\mathbb{B}})\Gamma(\widetilde{I} + \widetilde{\mathbb{B}}) \tag{A4}$$

and

$$\frac{d\widetilde{\mathbb{A}}}{dz} = v\widetilde{\mathbb{A}}\Gamma(\widetilde{I} + \widetilde{\mathbb{B}}), \tag{A5}$$

where $v=ik_o^2ag^2/\Lambda$ and the matrix Γ is defined by $\Gamma_{\mu\nu} = g^{-2}\gamma^{-1/2}(\mu)\gamma^{-1/2}(\nu)$. We seek a solution to the system of Eqs. (A4) and (A5) in a finite-dimensional complex linear space, with $|\mu|,|\nu|<\mu_{\max}$. In this linear space the matrix Γ is idempotent (see, e.g., Ref. [42]) having the property $\Gamma^2=\Gamma$. This property enables one to transform a solution of the matrix Riccati Eq. (A4) into a solution of a scalar Riccati equation that gives $\widetilde{\mathbb{B}}(z)=vz(1-vz)^{-1}\Gamma$ and $\widetilde{\mathbb{A}}(z)=\widetilde{I}+\widetilde{\mathbb{B}}(z)$. Taking here $z=h$ leads to Eqs. (29).

Some remarks are pertinent here concerning the resonant conditions (31). The cooperative scattering coefficient $\hat{b}$ is rewritten as

$$\hat{b} = \frac{2\pi}{\Lambda}\left[y(x) - \frac{i}{2x^{1/2}}\right]^{-1}, \tag{A6}$$

where $x=(\Lambda k_o/2\pi)^2<1$ and

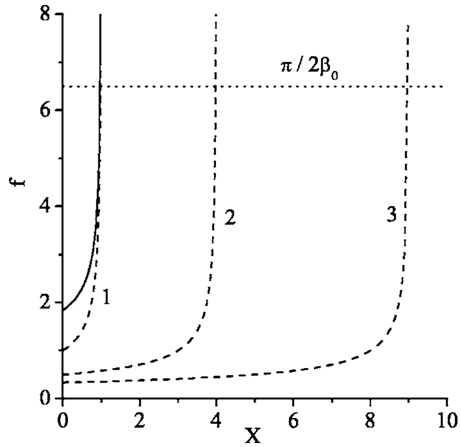

FIG. 5. Illustration of the graphic solution to the resonant equation $y(x)=0$ [see Eq. (A7)], dashed lines (1,2,3) correspond to $f=(\mu-x)^{-1/2}$ where $\mu=1,2,3$, respectively; solid line depicts the sum of these three functions; dotted line shows the constant $\pi/2\beta_o$.

$$y=\frac{\pi}{2\beta_o}-\sum_{\mu=1}^{\mu_{\max}}(\mu^2-x)^{-1/2}. \quad \text{(A7)}$$

The terms under sum in the RHS of the last equation and an illustration of the graphic solution to the resonant equation $y(x)=0$ are depicted in Fig. 5. Because of condition $\beta_o \ll 1$, the main contribution to the solution x_o of the resonant equation is given the first term with $\mu=1$ under the sum mentioned. Putting $\pi/2\beta_o \approx (1-x)^{-1/2}$, we obtain $x_o \approx 1-(2\beta_o/\pi)^2$ that gives a forbidden frequency in the radiation transmission spectra of a propagating wave through the 1D grating with linelike rulings presented before Eq. (32). The expansion of the function $y(x)$ into the Taylor series near the resonant point x_o, gives a resonant asymptotic for the cooperative scattering coefficient $\hat{b}$ in the form $\hat{b}\approx 2ik_o[1-i(k_o-k_{oo})/\Delta k_o]^{-1}$ where $k_{oo}\approx 2\pi/\Lambda$ and the resonant width Δk_o is also presented before Eq. (32).

APPENDIX B: DISTANT-SPATIAL SPECTROSCOPY OF EVANESCENT WAVES

Consider the second Eq. (33) in the limit $L\to\infty$. Bearing in mind that $\gamma_1<\gamma_2$, one can obtain directly three following limit relations:

$$\lim_{L\to\infty}\exp(2\gamma_1 L)I(L|\text{ev})=|E_1|^2, \quad \text{(B1)}$$

and

$$\lim_{L\to\infty}\exp[(\gamma_2+\gamma_1)L][I(L|\text{ev})-\exp(-2\gamma_1 L)|E_1|^2]=\Gamma(2,1), \quad \text{(B2)}$$

and

$$\lim_{L\to\infty}\exp(2\gamma_2 L)\{I(L|\text{ev}) -\exp(-2\gamma_1 L)|E_1|^2-\exp[-(\gamma_2+\gamma_1)L]\Gamma(2,1)\}=|E_2|^2. \quad \text{(B3)}$$

In the case of three, four, etc., evanescent spectral orders taken into account, the limit relations (B1) and (B2) are not changed but the relation (B3) is changed in part because of inequality $\gamma_1+\gamma_3<2\gamma_2$.

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